Non-stationary excitation of two localized spin-wave modes in a nano-contact spin torque oscillator

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We measure and simulate micromagnetically a framework based upon a nano-contact spin torque oscillator where two distinct localized evanescent spin-wave modes can be detected. The resulting frequency spectrum is composed by two peaks, corresponding to the excited modes, which lie below the ferromagnetic resonance frequency, and a low-frequency tail, which we attribute to the non-stationary switching between these modes. By using Fourier, wavelet, and Hilbert-Huang transforms, we investigate the properties of these modes in time and spatial domains, together with their spatial distribution. The existence of an additional localized mode (which was neither predicted by theory nor by previous numerical and experimental findings) has to be attributed to the large influence of the current-induced Oersted field strength which, in the present setup, is of the same order of magnitude as the external field. As a further consequence, the excited spin-waves, contrarily to what usually assumed, do not possess cylindrical symmetry: the Oersted field induces these modes to be excited at the two opposite sides of the region beneath the nano-contact.

I. INTRODUCTION

Spin wave dynamics induced by spin transfer torque has received great attention in recent years.1,2 Devices where such dynamics can be observed are generally classified as spin torque oscillators (STOs) and can be realized using several different materials and geometrical configurations.3–11

Among those geometrical configurations, nano-contact STOs, where the current is injected via a nano-sized electrical contact into a micrometer-sized metallic mesa, are of particular interest from several points of view. In fact, in these devices, the excited spin waves can propagate and be damped out before experiencing scattering at the mesa boundaries. Hence, they represent a model case which allows to study the fundamentals of spin wave dynamics as well as to explore further practical applications, especially in the emerging field of magnonics.12–14

In recent works,15,16 some important details of spin wave modes excitation in nano-contact STOs were pointed out. In particular, it has been found that below a certain critical out-of-plane angle θc, two distinct spin wave modes could be excited and observed as “coexisting” in the frequency spectrum. However, theoretical investigations carried out on the properties of these modes revealed that while the blue-frequency shifting mode lying above the ferromagnetic resonance frequency (FMR) has a propagating nature the red-frequency shifting one lying below the FMR spectrum is a localized “bullet” mode.17,18 Micromagnetic simulations at T = 0 K and the corresponding wavelet analysis allowed to detect a persistent periodic switching between these spin wave modes (features which typically cannot be resolved experimentally by the slow integration dynamics of a spectrum analyzer). We also found that such a switching could be observed in simulations only when a sufficiently large current-induced magnetic (Oersted) field is included, hence allowing us to understand its fundamental importance in the dynamics of spin-torque oscillators.

In the present work, we demonstrate, both experimentally and numerically, the possibility to build up a framework where two localized modes coexist in the frequency spectrum. The key difference with respect to the setup of Ref. 16 is the lower value of the bias field. Because of that, a stronger competition between the current-induced (Oersted) and the external (Zeeman) fields takes place owing to their comparable amplitudes. By means of different advanced signal processing techniques (Fourier, wavelet, and Hilbert-Huang transforms), we have been able to extract relevant information on the features of these modes. In particular, we found that those spin-wave modes can coexist in the time domain being excited simultaneously.

II. DESCRIPTION OF THE EXPERIMENTAL SETUP AND NUMERICAL DETAILS

A. Experimental framework

The investigated nano-contact spin torque oscillator is composed of a pseudo-spin valve mesa [NiFe (4.5 nm)/Cu...
Brown’s equation for the “fixed” layer: \( \mathbf{p} \times \mathbf{H}_{\text{eff}} = 0 \) with no current. The LLGS equation is numerically solved by using our own 3D finite-difference time-domain micromagnetic code that employs a fifth-order Runge-Kutta integration scheme and a fixed time step of 80 fs. Simulations are restricted to a limited square computational region as large as \( L \times L \times d_{\text{FL}} = 1000 \times 1000 \times 4.5 \text{ nm}^3 \), by using a 2D mesh of discretization cells having sizes \( 5 \times 5 \times 4.5 \text{ nm}^3 \). As discussed in previous works, to avoid the spurious spin-wave reflection from the computational boundaries we implement abrupt absorbing boundary conditions based on an artificial increase of the damping parameter at the borders.

The other parameters used to simulate the current-induced spin-wave dynamics in the Permalloy FL are saturation magnetization \( \mu_0 M_0 = 1 \text{ T} \), spectroscopic Landé factor \( g = 2 \), damping constant \( \alpha = 0.01 \), spin-torque efficiency \( \varepsilon = 0.3 \), and exchange stiffness constant \( A_{\text{ex}} = 1.1 \times 10^{-11} \text{ J/m} \).

The parameters used to compute the equilibrium magnetic state of the Co-based PL are thickness 20 nm, saturation magnetization 1.88 T, and exchange stiffness constant \( 2.0 \times 10^{-11} \text{ J/m} \). Simulations are 30 ns long in order to ensure a spectral resolution of about 30 MHz.

C. Wavelet analysis

We analyzed the time domain traces as computed from micromagnetic simulation by means of the Wavelet Transform (WT). Given a time-domain signal \( x(t) \), the continuous wavelet transform is a linear function \( W(u, s) \) given by

\[
W(u, s) = \frac{1}{\sqrt{s}} \int_{-\infty}^{+\infty} x(t) \psi^* \left( \frac{t-u}{s} \right) dt,
\]

where \( s \) and \( u \) the scale and translation parameters of the mother wavelet \( \psi(\cdot) \) that defines the wavelet family function as \( \psi_{u,s}(t) = \frac{1}{\sqrt{s}} \psi \left( \frac{t-u}{s} \right) \). In our study, in order to identify the non-stationary behavior of the oscillator, we used the complex Morlet wavelet family

\[
\psi_{u,s}(t) = \frac{1}{\sqrt{\pi f_s}} e^{2\pi i f_c t} e^{-\pi \alpha (f_t)^2},
\]

where \( f_b \) and \( f_c \) are two constant parameters (which we set to \( f_b = 100 \) and \( f_c = 1 \)) and \( \mathcal{W}(f) = F \left( \psi(t) \right) \) is the Fourier transform of \( \psi(t) \). Given a fixed dimension \( N \) for the scale set \( \{s\} \), we adopted the method proposed in Ref. 28 to find the optimal scale set \( \{s_i\}_{i=1, \ldots, N} \) for the WT. The use of a wavelet analysis allows to characterize a signal in the time-frequency domain and to identify non-stationary behaviours.

D. Hilbert-Huang transform

Hilbert-Huang Transform (HHT) is a recently developed method which is proved to be a successful approach for studying the nonlinear behavior of time series. By means of this technique, complex sets of nonlinear, non-stationary datasets can be decomposed into a finite collection of individual characteristic oscillatory modes, named intrinsic mode functions (IMF), through a process known as Empirical Mode Decomposition (EMD). These IMFs have well defined instantaneous frequencies and are assumed to
represent the intrinsic oscillatory modes embedded in the original signal.

Let us recall the key mathematical points of this technique. HHT consists of two parts: Hilbert transform (HT) and EMD. Given a time-domain function \( x(t) \), its Hilbert transform \( y(t) = H\{x(t)\} \) is defined as

\[
y(t) = P \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{x(s)}{t-s} \, ds,
\]

where \( P \) is the Cauchy principal value. If \( z(t) \) is the analytic signal associated to \( x(t) \), we have, for all \( t \),

\[
z(t) = x(t) + iy(t) = A(t)e^{j\phi(t)},
\]

where \( A(t) = \sqrt{x^2(t) + y^2(t)} \) and \( \phi(t) = \arctan\left(\frac{y(t)}{x(t)}\right) \) are the instantaneous amplitude and phase associated to the signal, respectively. The instantaneous frequency \( \omega(t) \) is defined as the time derivative of \( \phi(t) \), \( \omega(t) = \frac{\text{d}\phi(t)}{\text{d}t} \), while the instantaneous power, \( P(t) = |A(t)|^2 \), reflects how the power of the signal \( x(t) \) varies with time. Hilbert analysis provides a method for determining the instantaneous power and frequency of a mono-component signal. A generalization of the notion of such an analytic signal to a multi-component one is possible by using the EMD method.

It is an adaptive and efficient method applied to decompose nonlinear and non-stationary signals. It extracts a series of IMFs from the analyzed signal by means of an iterative method which is known as sifting process. It can be summarized as follows: (i) starting with the original signal, \( x(t) \), set \( h_1(t) = x(t) \) \((i = 1, 2, \ldots)\), extract the local minima and local maxima from \( h_i(t) \); (ii) interpolate the local minima and local maxima with a cubic spline to form upper and lower envelopes respectively; (iii) obtain the mean of the upper and lower envelopes, \( m_i(t) \), and subtract it from \( h_i(t) \) to determine a new protomode function (PMF) \( h_{i+1}(t) = h_i(t) - m_i(t) \). The above procedure is repeated until \( h_{i+1}(t) \) satisfies the ending criteria (specified below) of an IMF and then \( c_j(t) = h_{i+1}(t) \), where \( j = 1, 2, \ldots, n \) is the \( j \)-th IMF component from the data.

An oscillating wave to be an IMF must satisfy two conditions: (a) the number of extrema and the number of zero-crossings differs only by one and (b) the local average is zero. To preserve the natural amplitude variations of the oscillations, sifting must be limited to the lowest number of mathematically permissible steps; in this sense, the choice of a proper stopping criterion is crucial. This method is iterated by removing each oscillating wave which meets the above conditions, until the negligible residue signal \( r(t) \) is not an IMF. If the \( n \) IMF components have been determined, the original signal can be reconstructed by using the HT as

\[
x(t) = \sum_{j=1}^{n} c_j(t) + r(t).
\]

The signal can be recast in terms of the Hilbert transform as

\[
x(t) = \text{Re}\{z(t)\} \approx \text{Re}\left[ \sum_{j=1}^{n} A_j(t) \exp\left(i \int \omega_j(t) \, dt \right) \right].
\]

where the residue \( r(t) \) is ignored. Equation (4) can be seen as a generalized form of the Fourier decomposition for the function \( x(t) \) where both amplitude \( A(t) \) and frequency \( \omega(t) \) are functions of time. This approach insures that the instantaneous amplitude and frequency of each component of the resulting signal have physical meaning.

It has to be emphasized that, by means of this tool we are able to perform a complete separation of the oscillation modes in the time domain and so it is possible to acquire a better understanding of nonlinear dynamics which take place in our system.

### III. RESULTS

#### A. Experiments

Fig. 1(a) shows the measured power spectral density in the frequency range 0.1–25 GHz for a fixed field of 4.6 kOe and for an electric sweeping from 5 to 15 mA, whereas in Fig. 1(b) the same quantity is measured for a fixed current of 15 mA and a magnetic bias field in the range between 3 and 6 kOe. A number of observations can be deduced from those experimental data. The power spectra clearly indicate the excitation of two modes for a wide range of field and current. Since the bias field is orientated at an out-of-plane angle well below the critical angle below which both typologies of modes (propagating and localized) can be excited (according to previous findings15–18), one could identify the low and the high frequency mode as the localized and the propagating spin wave mode, respectively. However, as it will be demonstrated, theoretical investigations will describe a completely different scenario.

Indeed, we do not expect the excitation of a propagating mode since, due to the relatively small intensity of the bias field (4.6 kOe) with respect to the saturation magnetization...
of the FL (10 kOe), and considering the field orientation (just 30° out-of-device-plane), the magnetization of the FL lies almost entirely in the plane of the device. According to previous findings,16 we believe that our operating regimes fall into the region where only localized “bullet” modes should be excited and, consequently, no propagating modes should be observed.

Before analyzing it in detail, let us describe, first, the composition of the measured spectra. In Fig. 1(a), for currents \( I \geq 10 \text{ mA} \), two relatively strong signals can be observed: a low frequency signal at around 14 GHz and a high frequency one at about 17 GHz, whose separation increases from approximately 1.5 GHz–3 GHz. This increasing separation with current is due to the fact that the lower frequency signal “red” shifts while the high frequency signal “blue” shifts with bias current respectively. A typical power spectrum (at \( I = 15 \text{ mA} \)) is displayed in the inset of Fig. 1(a).

Together with the two dominant modes, we also observe a low-frequency tail (\( \leq 1 \text{ GHz} \)) signal with a pronounced current dependence, whose magnitude increases as the current is swept from 5 mA to 10 mA, where the onset of the spin wave excitations is located, and then decreases as the current is further increased. Moreover, at currents \( I \geq 10 \text{ mA} \), it is possible to detect a broad (\( \sim 5 \text{ GHz FWHM} \)) signal centered at about 7.5 GHz, which is more clearly visible in the inset of Fig. 1(a). This latter mode has a rather strong dependence on the applied magnetic field as can be noted in Fig. 1(b). In particular its amplitude decays monotonically as the applied field is increased. By displaying the decay of the measured power along the dashed line introduced in the main panel of Fig. 1(b), we find out a decreasing linear trend in the logarithmic scale (see the inset of Fig. 1(a)). This decay indicates that the power of this signal decays exponentially as a function of the applied magnetic field. This is consistent with a thermally activated mode which sees an increasing barrier height as the magnetic field is increased. Modelling the power decay as \( P \sim \exp(-\mu_B \Delta H/k_B T) \), where \( \mu_B = 9.27 \times 10^{-24} \text{ J/T} \) is the Bohr magneton and \( k_B = 1.38 \times 10^{-23} \text{ J/K} \) is the Boltzmann constant, and setting \( \Delta H = 3 \text{ kOe} \), equal to the range of variation of the bias field, we obtain a very good agreement with the experimental decay rate computed at \( T = 350 \text{ K} \) (temperature that can be related to the local heating caused by the high current densities (\( \sim 10^8 \text{ A/cm}^2 \)) beneath the nano-contact). However, a systematic study of this mode and its relationship with the main exited modes will be presented elsewhere.

B. Micromagnetic simulations

Being the most relevant and interesting results obtained for fields smaller than 5 kOe, we will discuss now in more detail the data achieved at 4.6 kOe without loss of generality. Fig. 2(a) shows the frequency of the excited modes as function of the Oersted field \( (H_{\text{Oe}}) \) amplitude, with respect to that originated by a current flowing through an infinite wire \( (H_{\text{inf}}) \), for \( I = 15 \text{ mA} \). For \( H_{\text{Oe}} = 0 \), only the mode “B” is excited, and it is localized below the whole contact. As the \( H_{\text{Oe}} \) increases, a deformation of the spatial distribution of the mode takes place and, at \( H_{\text{Oe}} = 0.8 H_{\text{inf}} \) a second mode “A” at a lower frequency appears in the power spectrum.

We believe that the origin of this scenario lies in the relative weights between applied and Oersted fields. In order to estimate the ratio between the strengths of these fields, let us consider a current of \( I = 15 \text{ mA} \) and assume that, for simplicity, the current flows through a 100 nm-thick cylinder, mimicking the whole pillar stack. Under these conditions, our numerical code26 provides the value of about 1.5 kOe for the maximum of the Oersted field at the nanocontact edge. This value corresponds to about 1/3 of the external field strength (4.6 kOe). In the previous work,16 the applied field was approximately twice larger in magnitude than in the present work so that such a ratio was about 1/10. Hence, the inhomogeneities in the magnetization caused by the Oersted field have now a larger relative weight and may be the main responsible of the intriguing properties exhibited by these modes.

In order to establish their nature, it is fundamental to preliminary determine the FMR frequency for the present setup. Indeed, one key characteristic of propagating and localized modes is that their spectra lie above and below the FMR frequency, respectively. Analytically, from the theory developed in Ref. 29, the resulting FMR is 21.5 GHz. We have also verified it numerically, through a spin-transfer-torque-FMR technique making use of an ac current of 5 mA, and we get a value very close to 21 GHz, which is in good agreement with theory (see Fig. 2(b)). From the above analyses, we conclude that the frequency of both modes is below the FMR so that the wavevector associated to both of them is an imaginary quantity, and, thus, we are not dealing with propagating waves, but with evanescent ones.17,18

As it will be shown, the same conclusion about the localization of both modes can be achieved in other three different ways.
The first one is strictly related to the analysis of the results shown in Fig. 2(a). Indeed, in the absence of Oersted field, only the mode “B,” which lies below the FMR frequency, is excited. This mode, according to the investigations carried out in Ref. 18, corresponds to a localized “bullet” mode. When increasing the Oersted field strength, a second mode (mode “A”) having frequency smaller than that of mode “B” appears, so that a fortiori the mode “A” should have a localized character as well.

The second one is connected to the spatial distribution of the excited spin wave modes. Such information is retrieved by performing further post-processing on the results of simulations, as depicted in Fig. 3. In this figure we indicate the position of the nano-contact as a white empty circle and the direction of the current induce Oersted field as small arrows rotating around the nano-contact. The in-plane component of the applied magnetic field is represented by the larger arrow on the bottom part of the panel. It is evident that the two modes are excited in two regions with peculiarly different magnetic states. The localized mode “A” in Fig. 3(a) has its maximum excitation amplitude where applied field and Oersted field tend to cancel each other. On the contrary, the localized mode “B” in Fig. 3(b) is mostly excited in the region where the two fields add up constructively (which, in turn, determines the higher excitation frequency). Only a small portion of the energy of the modes is located outside the nanocontact region.

The localization of the modes can be also estimated by plotting, as in Fig. 4, the amplitude of the z-component of the magnetization along the x-direction. From it, we infer that the magnetization decay is of the same order of magnitude for the two modes and that, at a distance of about 5 times the contact radius, the amplitude is reduced by five orders of magnitude.

All these arguments definitively confirm the strong localization of both modes. We would like to remark that it is possible to rule out that such a mode localization is related to dissipation, considering that the FL material (Permalloy) exhibits low-damping ($\alpha = 0.01$) and that typical decay lengths in such materials are of the order of several microns.\(^{11}\) We can analogously exclude that the two localized modes are thermally-activated. Indeed, by switching-off the thermal field in our numerical simulations, these modes preserve the spatially distribution and the spectral properties described above.

We would also like to stress that the existence of these localized modes strictly depend upon the bias field strength and direction. In fact, by increasing the bias field strength, the two localized modes become suppressed. This result is consistent with our previous findings\(^ {15-18}\) where, as a consequence of a larger out-of-plane component of the equilibrium magnetization, two distinct modes (a localized and a propagating one), uniformly distributed over the whole nanocontact area but not simultaneously excited, are observed instead. The same situation occurs when the bias field is directed towards larger out-of-plane angles. In fact, such a configuration favors the excitation of propagating modes and the disappearance of localized modes. On the contrary, the results are independent on the azimuthal angle, for symmetry reasons.

C. Time domain analysis

In addition to the previous investigations, we performed a detailed time domain analysis of the micromagnetic time traces. Figure 5 shows the micromagnetic time trace (a) obtained at $H_{ext} = 4.6 \text{kOe}$ and $H_{Oe} = 0.8 \text{ H_{inf}}$ with $I = 15 \text{ mA}$.
and its WT (b). Our computations show that the switching
between the two modes does not occur in the same determin-
istic fashion as in the previous work. Indeed, different
regions of interest can be easily detected: (i) individual exci-
tation of the mode “A” (R1) or “B” (R3); (ii) simultaneous
excitation of both modes (R2); and (iii) absence of excitation
(R4).

This result also suggests us that the frequency tail
observed in the frequency spectrum (inset of Fig. 1(a)) has to
be related to a non-stationary telegraphic switching between
the two dominant modes.

To gain further insights into the spin-wave dynamics,
we performed an additional time domain study on the same
trace by using the HHT. To investigate the instantaneous
properties of the signals, the simulated data was first decom-
paced into a summation of modal components using the
EMD technique. In our simulations, we used the Cauchy-like
stopping criterion as described in Ref. 23, limiting the size of
the standard deviation computed from two consecutive sift-
ing results to $SD = 0.01$. Results, which are shown in Fig. 6,
confirm the successfully achieved separation of the two dom-
inant modes “A” and “B” (which are, respectively, the first
and the second IMF of the signal $x(t)$, in order of relevance).

In detail, in Figs. 6(a) and 6(c) we depict the time-
domain graphs of the two modes as resulting from the HHT
procedure for $H_{ext} = 4.6 \text{kOe}$ and $I = 15 \text{mA}$, while in (b), (d) we
show the corresponding Fourier spectra (including the
Lorentzian fits). These latter results validate the correctness
of the HHT procedure, since they reveal the existence of a
frequency peak at $14.7 \text{GHz}$ (Fig. 6(c)) and at $17.6 \text{GHz}$
(Fig. 6(d)), which correspond to the low-frequency “A”
mode and the high-frequency “B” mode, consistently with
the results of Fig. 1(b). It should be pointed out that, after
adopting this time-domain separation, the two modes appear
also well separated in frequency so that they can be consid-
ered as independent ones.

In Figures 6(a) and 6(c) it is also possible to notice some
time windows in which the modes are switched off and
regions where they appear well activated, in agreement with
results arising from the WT (Fig. 5(b)). However, it has to be
recalled that the key difference between HHT and WT is that
the former technique has a better capability to separate in the
time domain the signals attributed to different modes very
close in frequency (within an octave). For this reason, the
technique we have here developed can act as a non-linear fil-
ter which, applied to time-domain traces of STO in strong
non-linear regime, allows to identify and separate the excited
modes.

IV. CONCLUSIONS

In conclusion, we measured and simulated spin wave
excitations in nano-contact spin torque oscillators, revealing
the new occurrence of the excitation of two localized modes.
Indeed, none of these modes has the typical features of a
propagating mode: the frequency is below the FMR spec-
trum, the energy associated to the modes is confined in a
very reduced region close to the nanocontact area and the
associated wavevector is imaginary. The structure of the
low-frequency spectrum points out a non-stationary switch-
ing between these modes. Also, the wavelet analysis reveals
the existence of time windows characterized by excitation of
a single localized mode, absence of excitation, or even si-
multaneous excitation of both modes.

Remarkably, and contrarily to what previously thought,
spin waves excited by spin transfer torque in a nano-contact
spin torque oscillator can exhibit a non-cylindrical symmetry.
These features are the signature of the strong influence
of the Oersted field which is here of the same order of mag-
nitude of the external field.

In addition, through the use of the HHT tool, it has been
possible to efficiently decompose and identify the time traces
associated to these modes, in spite of them being within an
octave. Owing to the wide applicability of this technique, we
believe that it can be successfully exploited in other different
physical scenarios.

In our opinion, all these observations could also stimu-
late further the research in spin wave dynamics. In particu-
lar, they could provide useful insights for the optimal
design of future magnonic devices as well as for synchroni-
zation of multiple spin-wave modes excited within the
same device.
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8 D. Houssameddine et al., Nat. Mater. 6, 447 (2007).
17 A. Slavin and V. Tiberkevich, Phys. Rev. Lett. 95, 237201 (2005).