Synchronization of propagating spin-wave modes in a double-contact spin-torque oscillator: A micromagnetic study

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**A B S T R A C T**

This work tackles theoretical investigations on the synchronization of spin-wave modes generated by spin-transfer-torque in a double nano-contact geometry. The interaction mechanisms between the resulting oscillators are analyzed in the case of propagating modes which are excited via a normal-to-plane magnetic bias field. To characterize the underlying physical mechanisms, a multi-domain analysis is performed. It makes use of an equivalent electrical circuit, to deduce the output electrical power, and of micromagnetic simulations, through which information on the frequency spectra and on the spatial distribution of the wavefront of the emitted spin-waves is extracted. This study provides further and intriguing insights into the physical mechanisms giving rise to synchronization of spin-torque oscillators.

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1. Introduction

A spin-torque nano-contact oscillator (STNO) is a device constituted by a classical extended spin-valve layered structure, ferromagnet-metallic spacer-ferromagnet, on the top of which a metallic nano-contact allows for the flow of an electric current through a reduced volume of the magnetic structure. The lateral dimensions of such devices are in the order of tens of microns whereas the metallic contact is typically circular with a radius of about 20–100 nm. Several experimental and theoretical studies have been performed to determine the behavior of this kind of oscillators [1–12].

The most appealing features of nano-contact devices from the point of view of potential technological applications are tunability with both dc current and magnetic field, high quality factor [3] of the oscillations and nano-scale dimensions. On the other hand, low values of the output power, in the order of some pWs, have been experimentally measured [3,10] so that a large-scale technological applications are so far quite limited. To overcome this problem, the chance to make two or more interacting oscillators has been studied [13–24]. It has been shown, indeed, that their synchronization, achieved through a mutual phase-locking mechanism, increases the emitted power and, also, improves further the spectral purity [13,14]. Different geometries have been considered to connect more STNOs, such as serial circuits with the possibility of a feedback loop [24] or multiple metallic contacts on the same free magnetic layer [13–23]. In this latter case, which can be treated as a parallel connection, it has also been studied theoretically that two different types of interaction can be responsible for the synchronization between out-of-plane magnetized STNOs: magnetodipolar interaction and spin-wave propagation [22,23]. In particular, it has been shown that the former prevails for very short and very long distances, whereas the latter dominates in the intermediate range [15,22,23]. With this in mind, differently from the previously mentioned works, here we perform a numerical analysis of the synchronization dynamics occurring in a double-contact STNO subject to a normal-to-plane external magnetic field with the aim of highlighting the operating conditions under which synchronization takes place and extracting some key features as a function of the underlying physical mechanisms and of the distance between the contacts. The occurrence of synchronization will be demonstrated through a combined study which analyze the frequency spectra, the spatial maps of the wavefront of the excited spin-waves and the emitted output power.

2. Electrical modeling

We preliminary review a theoretical electrical model for the two oscillators connected in parallel. An STNO is usually biased and connected to a resistive load $Z_0$ as depicted in Fig. 1a. If the applied direct current $I_d$ has proper strength and direction, it can allow the excitation of a persistent precession of the FL magnetization at a frequency $f = \omega/2\pi$. The
inductance and capacitance in the circuit are generally inserted to decouple the dc currents from the microwave signal. Since the resistance of the structure is periodically changing, an ac voltage is thus obtained across the STNO. We can express this voltage as follows:

$$\nu_{ac}(t) = \Delta R_{max} I_{dc} \cos(\omega t + \phi)$$

(1)

being $\Delta R_{max}$ the maximum deviation of the resistance from the equilibrium value and $\phi$ the phase at time $t=0$. The overall voltage would also include a dc component due to the average internal resistance $R_i$ of the device but, being this resistance negligible with respect to both $\Delta R_{max}$ and the load, we disregard this contribution [24]. The resulting Thevenin-based equivalent electrical scheme [25] of the STNO connected to the load is shown in Fig. 1b. The voltage delivered to the load $Z$ can be thus considered equal to $\nu_{ac}$

$$v_L = \frac{Z_0}{Z_0 + R_i} v_{ac} \approx v_{ac}$$

(2)

and the active power delivered to the load is

$$P_L = \langle \nu_{ac}^2 \rangle = \frac{\Delta R_{max}^2 I_{dc}^2}{Z_0} \langle \cos^2(\omega t + \phi) \rangle = \frac{1}{2} \frac{\Delta R_{max}^2 I_{dc}^2}{Z_0}$$

(3)

where the notation $\langle \rangle$ denotes the mean value.

By denoting $i_{ac}$ the mesh current in Fig. 1b and by using the substitution theorem [25] and the above assumptions, the previous circuit can be also schematized in terms of an ac current source

$$i_{ac} = \frac{\Delta R_{max} I_{dc}}{Z_0} \cos(\omega t + \phi)$$

(4)

as shown in Fig. 1c. The power delivered to the load can be equivalently expressed as follows:

$$P_L = \langle Z_0 i_{ac}^2 \rangle = Z_0 \frac{\Delta R_{max}^2 I_{dc}^2}{Z_0} \langle \cos^2(\omega t + \phi) \rangle = \frac{1}{2} \frac{\Delta R_{max}^2 I_{dc}^2}{Z_0}$$

(5)

Let us now consider a double-contact STNO system, in which the contacts are connected to two independent dc current sources as in Fig. 2a. The corresponding electrical model (Fig. 2b) includes two equivalent ac current sources

$$i_{ac1} = \frac{\Delta R_{max} I_{dc1}}{Z_0} \cos(\omega_1 t + \phi_1), \quad i_{ac2} = \frac{\Delta R_{max} I_{dc2}}{Z_0} \cos(\omega_2 t + \phi_2)$$

(6)

where the internal resistances of the two oscillators have been again neglected with respect to the load.

The current that goes to the load, $i_L$, is given by

$$i_L = i_{ac1} + i_{ac2}$$

(7)

whereas the delivered power is

$$P_L^{(2)} = Z_0 \langle i_L^2 \rangle = Z_0 \langle (i_{ac1} + i_{ac2})^2 \rangle$$

$$= \frac{1}{2} \frac{\Delta R_{max}^2 I_{dc1}^2}{Z_0} + \frac{1}{2} \frac{\Delta R_{max}^2 I_{dc2}^2}{Z_0} + \frac{\Delta R_{max}^2 I_{dc1} I_{dc2}}{Z_0} \times \left( \langle \cos (\omega_1 + \omega_2) t + (\phi_1 + \phi_2) \rangle \right)$$

$$+ \left( \langle \cos (\omega_1 - \omega_2) t + (\phi_1 - \phi_2) \rangle \right)$$

(8)

By comparing the first two terms of the last expression with Eq. (5), we note that they represent the powers delivered to the load by each contact, $P_{L1}$ and $P_{L2}$. We can thus rewrite Eq. (8) as

$$P_L^{(2)} = P_{L1} + P_{L2} + 2 \sqrt{P_{L1} P_{L2}} \left( \langle \cos (\omega_1 + \omega_2) t + (\phi_1 + \phi_2) \rangle + \langle \cos (\omega_1 - \omega_2) t + (\phi_1 - \phi_2) \rangle \right)$$

(9)

Let us now consider three different cases

- non-synchronization: $\omega_1 \neq \omega_2$.
  In this case $\langle \cos (\omega_1 + \omega_2) t + (\phi_1 + \phi_2) \rangle = \langle \cos (\omega_1 - \omega_2) t + (\phi_1 - \phi_2) \rangle = 0$ and the delivered power simply turns out to be the sum of the powers delivered by the two contacts

$$P_L^{(2),ns} = P_{L1} + P_{L2}$$

(10)

- out-of-phase synchronization: $\omega_1 = \omega_2 = \omega$ and $\phi_1 \neq \phi_2$.
  Eq. (9) becomes

$$P_L^{(2),os} = P_{L1} + P_{L2} + 2 \sqrt{P_{L1} P_{L2}} \cos (\phi_1 - \phi_2)$$

(11)

The previous expression indicates that the output power depends on the phase difference between the two oscillations. In particular, it implies that, in spite of synchronization takes place, the output power could be even less than the sum of the
output powers of the two contacts if the condition $90^\circ < |\phi_1 - \phi_2| < 270^\circ$ holds.
- in-phase synchronization: $\omega_1 = \omega_2 = \omega$ and $\phi_1 = \phi_2 = \phi$.

The output power becomes
\[ P^{(2)_{\text{in}}} = P_{11} + P_{22} + 2\sqrt{P_{11}P_{22}} \] (12)

This is the highest possible value achievable in the configuration of two synchronized STNOs. Moreover, if the contacts are biased by the same direct current $I_{dc}$ and present the same $\Delta R_{\text{max}}$, the output powers from the two contacts are the same, $P_{11} = P_{22} = P_t$, and the overall power delivered to the load becomes four times the power delivered by a single STNO,
\[ P^{(4)_{\text{in}}} = 4P_t. \] This latter result can be easily generalized to the case of $N$ interacting STNOs where the maximum achievable power scales with $N^2$.

3. Micromagnetic modeling

Dynamics excited in a double-contact STNO are numerically studied by means of micromagnetic simulations. Let us describe, first, the micromagnetic model of the device and the setup here considered.

The two metallic circular contacts of the above-described STNO are of radius $R_{C1}$ and $R_{C2}$ and are lithographically defined on the top of the FL at positions $r_1$ and $r_2$ with respect to the center of the FL where $r=0$, providing the opportunity to apply a perpendicular-to-plane current in reduced regions of the FL only (see Fig. 3).

The dynamics of the magnetization vector of the FL in both time and spatial domain, $M = M(t,r)$, is governed by the Landau–Lifshitz–Gilbert–Slonczewski (LLGS) Eq. (1)
\[ \frac{dM}{dt} = \gamma [H_{\text{eff}} \times M] + \alpha \left[ \frac{M \times \partial M}{\partial t} \right] + \left[ f_1(r, r_1, R_{C1}) \frac{\sigma_1 I_{dc1}}{M_0} + f_2(r, r_2, R_{C2}) \frac{\sigma_2 I_{dc2}}{M_0} \right] [M \times (M \times p)] \] (13)

where $\gamma$ is the gyromagnetic ratio and $H_{\text{eff}}$ is the effective magnetic field which includes magnetostrictive, exchange, and Zeeman contributions. For simplicity, in our model we neglect the current-induced (Oersted) magnetic field, the magnetostatic coupling between the two ferromagnetic layers and thermal fluctuations as, in our opinion, they do not affect the wave features under investigation. We also ignore the magnetocrystalline anisotropy in the FL, which is an usual assumption for magnetically soft Permalloy layers. The second term in the right-hand side of Eq. (12) is the phenomenological magnetic damping torque written in the traditional Gilbert form ($\alpha$ is the damping constant) and $M_0 = |M|$ is the saturation magnetization of the FL. The last term is the Slonczewski spin-transfer torque that is proportional to the bias currents $I_{dc1}$ and $I_{dc2}$. The functions $f_1(r, r_i, R_{C_i})$ (where the subscript $i=1,2$ stands for the label of the nano-contact) describe the spatial distribution of the currents across the area of the nano-contacts. In the simplest case of uniform current density distributions, $f_i(r, r_i, R_{C_i}) = 1$ if $|r - r_i| < R_{C_i}$ and $f_i(r, r_i, R_{C_i}) = 0$ otherwise. The coefficients $\sigma_i$ are related to the dimensionless spin polarization efficiency $\epsilon_i$ by $\sigma_i = \epsilon_i g_i \mu_i / 2eM_0 S_i d_{\text{eff}}$, where $g_i$ is the spectroscopic Landé factor, $\mu_i$ is the Bohr magneton, $e$ is the absolute value of the electron charge, $d_{\text{eff}}$ is the FL thickness and $S_i = \pi R_i^2$ are the nano-contact areas. The unit vector $p$ defines the spin-polarization direction which coincides with the equilibrium direction of the PL magnetization. It is obtained by solving Brown’s equation for the FL: $p \times H_{\text{eff}} = 0$ with no current.

In our approach, the LLGS equation is numerically solved by using our own three-dimensional finite-difference time-domain (FD-TD) micromagnetic code that employs a fifth-order Runge–Kutta integration scheme [26]. Simulations are restricted to a limited square computational region as large as $L \times L \times d_{\text{fl}} = 1000 \times 1000 \times 4 \text{ nm}^2$, by using a 2D mesh of discretization cells having sizes $5 \times 5 \times 4 \text{ nm}^2$. As discussed in previous works [6,7,11,12], to avoid the spurious spin-wave reflection from the computational boundaries we implement abrupt absorbing boundary conditions based on an artificial increase of the damping parameter at the borders.

In order to simulate the natural differences between the contacts due to the realization processes of the samples, we choose slightly different parameters for the radii, $R_{C1} = 20 \text{ nm}$ and $R_{C2} = 25 \text{ nm}$, and for the spin-polarization efficiencies, $\epsilon_1 = 0.30$ and $\epsilon_2 = 0.25$, of the contacts. Contacts are positioned on the $x$ axis at the points $C_1 = (-r_1, 0)$ and $C_2 = (r_2, 0)$ and their distance $\Delta r = r_2 - r_1$ is set equal to 100 and 450 nm to distinguish, compatibly with the size of our computational domain, the behavior of close and distant contacts, respectively (see Fig. 3b).

The other parameters used to simulate the current-induced spin-wave dynamics in the Permalloy FL are: thickness $d_{\text{fl}} = 4 \text{ nm}$, saturation magnetization $\mu_0 M_0 = 0.88 \text{ T}$, spectroscopic Landé factor $g = 2.0$, damping constant $\alpha = 0.01$, and exchange stiffness constant $A_{\text{ex}} = 1.3 \times 10^{-11} \text{ J/m}$, strength of the external bias field $\mu_0 H_{\text{ext}} = 1 \text{ T}$.

The parameters used to compute the equilibrium magnetic state of the Co-based PL are: thickness $20 \text{ nm}$, saturation magnetization $1.88 \text{ T}$ and exchange stiffness constant $2.0 \times 10^{-11} \text{ J/m}$.

4. Numerical results

The numerical investigation is carried out as follows. We first model the behavior of each contact by switching off the current across the other. After that, we investigate the interaction between them by fixing the current at one contact and varying the current across the other one. The study is performed for close and distant contacts. During the post-processing of the resulting data, we analyze the phenomenon responsible for the synchronization in
both frequency and spatial domains and, also, evaluate the output power by comparing the obtained numerical value with the one predicted by the previous electrical model.

Let us describe first the results of the individual characterization of each contact. Fig. 4a shows the corresponding output frequencies $f_1$ and $f_2$ from the two contacts for a perpendicular-to-plane magnetic field. For simplicity, we limit our analysis to the interval of current 14–34 mA that corresponds to a range of frequencies of about 14–24 GHz. As it is expected for this inclination of the field, the modes excited below the contacts propagate towards the borders of the FL, and the frequencies increase with the current. The choice of different parameters for the contacts leads to slightly different curves as shown in Fig. 4a, being the linear threshold current of contact 1 lower than that of contact 2. The slopes of these two curves are, on the contrary, quite similar, since the effects of the different contact radii and spin polarization efficiencies compensate each other [22].

Once the behavior of each contact is known, then we allow the current to flow through both contacts simultaneously. Being independent, we choose to vary $I_{dc2}$ and to fix $I_{dc1} = 14$ mA, which corresponds to the excited frequency $f_1 \approx 20$ GHz, which is in the middle of the range of frequencies observed for the second contact. Fig. 4b and c shows the output frequencies read under the contacts, as a function of $I_{dc2}$, in two different situations: close ($\Delta r = 100$ nm) and distant ($\Delta r = 450$ nm) contacts.

In the former case, $f_1$ and $f_2$ exactly overlap to each other, pointing out that synchronization takes place in the entire range of the current $I_{dc2}$ (see Fig. 4b). In addition, the synchronization frequency remains between the values of the individual frequencies obtained during the first stage (Fig. 4a). As it will be clear later in the text, for each value of the current $I_{dc2}$, such an intermediate value of the frequency of the corresponding synchronized mode is the result of the interaction arising between the spin wave modes.

In the case of distant contacts, however, Fig. 4c shows three different regions. For $I_{dc2} < 22$ mA no synchronization takes place. Indeed, $f_1$ and $f_2$ are different and correspond, except for a small difference that pushes them a little closer, to the frequencies obtained in the case of individual excitation (Fig. 4a). In other words $f_1$ remains quite constant as there is no change of $I_{dc1}$, while $f_2$ increases with $I_{dc2}$. The range 22 mA $\leq I_{dc2} < 28$ mA is characterized by the synchronization phenomenon. Here, $f_1$ and $f_2$ coincide and, in particular, $f_1$ follows the trend of $f_2$, namely the frequency excited in the larger contact. Finally, for $I_{dc2} > 28$ mA, the two curves diverge as $f_1$ and $f_2$ get back to their unsynchronized values.

The next step is to distinguish which phenomenon is responsible for the synchronization between the oscillators: spin-wave coupling or magnetodipolar interaction. With this aim, we perform further numerical investigations where the free layer region between the metallic contacts is cut [15]. By means of this strategy, we are artificially breaking the magnetic continuity of the film. In particular, since exchange interactions are short ranged (only first neighbours), the discontinuity precludes interaction between the two contacts via spin waves. In this case, in fact, synchronization, if present, can only depend upon magnetodipolar interactions. Simulations are carried out with current values for which, in the case of continuous free layer, frequency locking is observed. In particular, we choose $I_{dc2} = 30$ mA for close contacts and $I_{dc2} = 26$ mA for distant contacts.

In the case of close contacts, a single frequency peak is observed in the output spectrum, which reveals the occurrence of synchronization. Since it cannot be due to spin-wave coupling, we can assert that synchronization between close contacts is mainly caused by magnetodipolar coupling. If we look in detail at the frequency spectra (not shown), we also notice a decrease of the output frequencies emitted by the two contacts of about 2 GHz. This occurrence has to be attributed to the concurrent decrease of the out-of-plane component of the effective field due to the magnetostatic interactions induced by the other contact.

Simulation performed with distant contacts on the cut FL shows, instead, that no synchronization takes place. Clearly, we can state that synchronization between distant contacts is mainly caused by the spin-waves that propagate in the FL, as the removal of the FL continuity has in turn destroyed the interaction between the spin-wave modes.

It should be noticed that, in spite of the use of a simplified framework neglecting the current-induced Oersted field, our results agree qualitatively well with the ones described in other experimental [13] and numerical [17] studies and, at the same time, provide the numerical confirmation of the theory given in [22,23].

In Fig. 5 we illustrate the results of this investigation by showing the micromagnetic spectral maps [27,28] corresponding to four different cases: distant contacts and continuous FL (Fig. 5a),
close contacts and continuous FL (Fig. 5b), distant contacts and cut FL (Fig. 5c), close contacts and cut FL (Fig. 5d). In order to better appreciate the occurrence of synchronization, we have added some guides to the eye, corresponding to wavefronts emitted from each contact. By means of this strategy, only in the first case it is possible to see a clear constructive interference pattern between the waves propagating from the contacts, as a mark of the synchronization due to spin-wave coupling. In Fig. 5b, though a more difficult interference pattern is detected (due to the close proximity of slightly different contacts), it is possible to notice a quite continuous wave-front in the region far from the contacts which resembles the behavior of a single wave source. This last consideration can be also made for the case of Fig. 5d, where the FL is cut and wave propagation is prevented. In these two cases, therefore, synchronization still takes place because of the magnetodipolar field. In Fig. 5c, instead, no sign of constructive interference is present, and synchronization does not occur, as the magnetodipolar coupling is too weak at long distances and spin-wave coupling is prevented by the cut in the FL. From the analysis of these results, we can conclude that the mark of synchronization due to spin-wave propagation is the classical pattern of two waves interacting constructively, whereas the coupling due to magnetodipolar interaction acts as a single wave source were present.

A further evidence of the occurrence of synchronization comes from the comparison of the output power resulting from the simulations and the electrical model developed in Section 2. Simulations can provide an estimation of the power through the height of the frequency peak in the signal spectrum. We therefore collected the power values both for the contacts acting separately (\(P_{L1}\) and \(P_{L2}\)), and for the case in which the contacts are simultaneously traversed by current (which gives rise to an overall signal of power \(P_{L,\text{NUM}}\)). From the simulations we can also determine the phase difference between the signals \(\Delta \phi = |\phi_1 - \phi_2|\) that has to be used, together with \(P_{L1}\) and \(P_{L2}\), to evaluate the power \(P_{\text{out}}\) as predicted by the model through Eq. (11). Fig. 6 summarizes such a comparison for both close (Fig. 6a and b) and distant (Fig. 6c and d) contacts, in the case of a continuous FL. In the same figure we also show the values of \(\Delta \phi\), in the range of currents where synchronization has been detected. In particular, when the contacts are closedly placed, \(\Delta \phi\) decreases with the current reaching a minimum and then increases (Fig. 6a). The simple sum of the individual powers, \(P_{\text{out}}\), increases as the current \(I_{dc2}\) increases. The overall power obtained by means of simulations, \(P_{L,\text{NUM}}\), agrees with the predictions of the model, \(P_{\text{out}}\), so confirming the occurrence of an out-of-phase synchronization phenomenon (Fig. 6b). Such an overall power also turns out to be higher than the sum of the individual powers for all the current values, as the phase difference \(\Delta \phi\) is between 0° and 90°. On the other hand, when the contacts are distant, \(\Delta \phi\) has a minimum at \(I_{dc2} = 26\,\text{mA}\), whereas the other values are higher than 90° (Fig. 6c) [9]. This latter situation, in spite of the presence of synchronization, leads to an overall output power which is smaller than \(P_{\text{out}}\). In fact, \(P_{\text{out}}\) turns higher than \(P_{L,\text{NUM}}\) only for \(I_{dc2} = 26\,\text{mA}\). The numerically computed power and the one predicted by the model are once again in agreement, pointing out the occurrence of out-of-phase synchronization.

5. Conclusions

Our study confirms the results of previous theoretical and experimental investigations and in particular reveals that synchronization between closely-positioned contacts is dominated by magnetodipolar interactions, whereas spin-wave coupling is the main responsible for distantly-positioned contacts. However, our study provides additional information arising from the spatial distribution of the wavefronts of the corresponding modes. Indeed, this investigation has allowed to argue that the synchronization via spin-wave coupling is characterized by the typical constructive interference pattern of two distinct wave sources. On the contrary, the unique wavefront resulting from magnetodipolar interactions leads to the conclusion that, in the synchronized regime, the system behaves as if a single wave source were present in the investigated region.
Finally, results from the equivalent electrical circuit which models the output power in the presence/absence of synchronization satisfactorily agree with those arising from micromagnetic simulations and thus constitute an additional tool to evaluate the occurrence of the locking mechanism between the wave sources.

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Fig. 6. Comparison between numerical results and the electrical model of a double STNO for (a) and (b) close and (c) and (d) distant contacts in the case of a perpendicular field. In (a) and (c) the phase difference between the waves emitted by the synchronized contacts is shown, (b) and (d) show the comparison between the simple sum of the two powers $P_L^{(2),ns}$, the power numerically obtained $P_L^{\text{NUM}}$, and the power predicted by the model for synchronized contacts $P_L^{(2),os}$. 

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