About Identification of Scalar Preisach Functions of Soft Magnetic Materials

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The paper shows the equivalence between algorithms that use different experimental data in order to identify the Preisach probability function. Furthermore, it is proposed an identification procedure which increases the computation accuracy of the numerical Preisach probability function of magnetic materials and reduces the necessary experimental data to solve the identification problem. The proposed approach uses a set of symmetric minor loops and a new discrimination function of the Preisach plane.

Index Terms—Hysteresis, numerical optimization, Preisach model, soft magnetic materials.

I. INTRODUCTION

The hysteresis is a complex nonlinear problem because it is due to existence of different effects (i.e., lag, dissipation, memory [1], [2]) at the same time. It is a key point to model the behavior of ferromagnetic materials used in many technological applications: sensors, electrical machines, storage devices, etc. [2], [3]. Several models have been proposed to describe the ferromagnetic hysteresis, in particular the Preisach model has been used with success in a lot of applications.

In the Preisach model, the main point is the identification procedure of the Preisach probability function (PPF). It can be solved by means of the knowledge of experimental first-order reversal curves (FORC) [1] or symmetric minor loops (SML) [4]. Given that manufacturers and users of soft magnetic materials, as indicated by the specific standards, require a set of SML and the first magnetization (virgin) curve in order to characterize physical properties of the materials under investigation, the FORC algorithm could be not always employed. Furthermore, from the experimental point of view, the SML measurement is generally carried out simpler than the FORC one.

In this paper, we point out about the scalar Preisach model of static hysteresis. In particular, we briefly discuss about the equivalence between two different identification procedures. Furthermore, we introduce an optimization of the SML procedure using a new discrimination function of the Preisach plane (PP). Lastly, a comparison between experimental and computed data will be shown.

II. EQUIVALENCE BETWEEN ALGORITHMS

In this section, we discuss about the equivalence between the FORC and the SML procedures if the material exhibits a symmetric hysteresis [2, pp. 444–448], the wiping-out and congruency properties [1, pp. 18–26].

The symmetric property is taken into account in the PPF as \( P(U, V) = P(-U, -V) \), where \( P(U, V) \) is the density function of hysterons, whereas \( U \) and \( V \) are the switching-up and switching-down fields respectively [3].

The equivalence can be demonstrated analytically using a Lorentzian–Lorentzian PPF [5]. For other distributions the equivalence can be proved numerically. In this paper, we show the numerical comparison in the Gaussian–Gaussian case [3]:

\[
P(U, V) = A \exp \left\{ -\frac{1}{2} \left[ \frac{(U-V-2H_C \sigma)^2}{2H_C^2} + \frac{(U+V-2H_C \sigma)^2}{2H_C^2} \right] \right\}
\]

where \( H_C \) and \( \sigma \) are the characteristic parameters of this distribution and \( A \) is a normalization factor. Fig. 1 shows the PPFs computed for both the FORC and the SML algorithms. In this case we found identical functions. The same results we have found for different PPFs.

III. IDENTIFICATION PROCEDURES

This section introduces an optimization of the SML identification procedure. The starting point of this analysis is described in [4], where the PPF is computed solving a linear system. The magnetic field \( H(t) \) starts from 0, it goes to the positive value \( k \Delta H \), then to the negative value \( -k \Delta H \), where \( k \) is a positive integer (1, ..., 6), and \( \Delta H \) is the step used to increase the applied field along the virgin curve (\( N \Delta H \) gives the saturation field \( H_S \)). For this reason, we refer to this procedure as “\( \Delta H \)-fixed approach” (HFA). This procedure employees an uniform discrimination function that subdivides the PP in equal size subintervals, namely every point in the PP has the same weight in the identification procedure, included the not physical ones (\( V > U \)).

The key idea of the new procedure is to use a different discrimination function in order to reduce the number of SML necessary to identify the PPF or, similarly, to increase the accuracy of the numerical PPF using the same number of SML. To do this, the new discrimination function has to assign variable relevance degrees to the PP regions, giving more importance to the physical ones. For example, if we suppose that the magnetic material under investigation is mainly single-quadrant, we can detect more information (that means a good accuracy) if most of points are located in the fourth quadrant of the PP.
The proposed approach subdivides the range between negative and positive saturation magnetization in subintervals using a fixed gap $\Delta M$ among them. The applied field values are obtained at the intersections among these subdivisions and the ascending branch of major loop. These represent the starting points of the SML along the virgin curve and, at the same time, the subdivisions of the U-axis of the PP (the ones of the V-axis are obtained by symmetry). The zero applied field value, if not included, has to be added. For these reasons, we refer to the new procedure as “$\Delta M$-fixed approach” (MFA). Fig. 2 shows the PP in the two approaches using a grid of $20 \times 20$ points.

In this way, it is possible to have more useful information since SML are more uniformly distributed in the $H$–$M$ plane than in the HFA case. Using the Lorentzian–Lorentzian as analytical approximation of the PPF and the same grid size of the PP ($40 \times 40$ points), the comparison between HFA and MFA leads to the results showed in Fig. 3. The mathematical formulation of the MFA is reported in the Appendix.

IV. EXPERIMENTAL VALIDATION

In order to carry out the experimental validation, the MFA approach has been tested on several magnetic materials. In particular, we present here results obtained using a Co$_{21}$Fe$_{27}$Nb$_7$B$_{15}$ cast alloy. Again, in both procedures we used the same grid size and density of the PP. Figs. 4 and 5 show the comparison between the PPF’s and the corresponding SML computed in both cases.

It is important to underline (see also the Appendix) that in the MFA case the subdivision cells have not the same geometrical shape. So, in this numerical comparison, the PPF’s values have been assigned to the center of each cell.

Results show a significant improvement of the identification accuracy of the PPF and a better agreement in calculated loops using the MFA with respect to the HFA.

V. ABOUT INTERSECTION BETWEEN LOOPS

The aim of this section is to demonstrate how an intersection occurring between SML yields negative values of the PPF which does not correspond to any physical meaning.

In order to show this, it is sufficient to consider two SML as in Fig. 6. Starting from the demagnetized state (point O), we increase the applied field until the field $H^*$ is reached (point A). Now, starting from the negative saturation state (point B) we increase the applied field until the same field $H^*$ is reached (point $A'$). If we get into insight the hysterons distributions obtained in both cases (showed in the same figure), we can note that, in order to reach the positive saturation state, in the first case (O-A) it is necessary the switch of an amount of hysterons (proportional to the area of the negative-state region) less than the one required in the second case (B-$A'$), in spite of the change
in magnetization is bigger in the first case \((\Delta M > \Delta M')\). This could be only justified if the PPF can assume negative values, but this statement has not a physical meaning.

VI. CONCLUSION

In this paper, we presented a comparison between two different numerical identification procedures of the PPF: the first one uses FORC while the second uses a set of SML. After a brief discussion about their equivalence under a few hypothesis, we have introduced an optimization of the SML procedure. In fact, the algorithm which uses an uniform subdivision of the PP (HFA) has the evident drawback to locate about half of points of the discrimination function in the region of the PP where \(V > U\). So, we have developed a new procedure to increase the accuracy in the numerical PPF identification, placing the points used for the identification mostly in the region \(U > V\). This procedure uses a new subdivision of the PP based on the MFA.

In order to evaluate the increment of the accuracy, the best case is taken into account. If we consider a grid size of \(n \times n\) points, the optimal discrimination function allocates in the unphysical region \(2n^2\) points only (the first row, \(V = +H_S\), and the first column, \(U = -H_S\), of the PP). In this way, there are \(n^2 - 2n\) points in the physical region in the MFA case and \(n^2/2\) points (always) in the HFA case. The difference between the accuracy obtained using the MFA with respect to the HFA can be summarized as follows:

\[
\frac{(n^2 - 2n) - \frac{n^2}{2}}{n^2} = 1 - \frac{2}{n}
\]  

(2)
It means we can obtain an accuracy improvement up to 50% (for $n \to \infty$), without increasing the computational complexity.

**APPENDIX**

Starting from [4], we get into insight the mathematical formulation of the MFA. In the following discussion, we consider, for the sake of simplicity, a few SML only, but results have a general validity. The PP has been discretized as in Fig. 7, where we have considered the PPF to be constant in each cell of the subdivision.

We indicate as $P_{(j,k)}$ the PPF at the $j$th field in which the positive $U$ axis is subdivided, and at the $k$th iteration step for that field. For example, as we can see in Fig. 7, the first increment of the applied field involves only two steps ($P_{11}$ and $P_{12}$), the second one three steps ($P_{21}, P_{22}, P_{23}$) and the third one five steps ($P_{31}, P_{32}, P_{33}, P_{34}, P_{35}$).

According to the classical scalar Preisach model, the magnetization formulation for a decreasing applied field is

$$M_{\text{initial}} - M_{\text{final}} = 2 \int \int P(U,V) dU dV$$

where $M_S$ is the saturation magnetization.

Then, starting from the first increment of the applied field ($H_ip_{j1}$), the change of the magnetization, integrating in the corresponding cell, can be expressed as

$$\frac{M_{11} - M_{12}}{2M_s} = \frac{1}{2} P_{11}H_ip_{j1}^2$$

and so on for the successive steps.

The general result is that each change of the magnetization, normalized to $2M_S$, is equal to the sum, extended to the PP subdivisions belonging to the same row at left of the current one (see Fig. 7), of the PPF value of the considered cell multiplied by the corresponding integration area. In this way, we can introduce the formulation shown at the bottom of the page.

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